J.K. SHAH CLASSES

MATHEMATICS & STATISTICS

FYJC TEST

SOLUTION

DURATION – $1^1/_2$ HR MARKS – 25

Q1. Attempt ANY TWO of the following (06)

01. A(3,5) and B(4,1) . Find the locus of point P such that $l(AP)^2 + l(BP)^2 = 60$

SOLUTION :

let P(x,y) be any point on the locus , A(3, 5) ; B(4,1)

As per the given condition

$l(AP)^2 + l(BP)^2 = 60$

$(x-3)^2 + (y-5)^2 + (x-4)^2 + (y-1)^2 = 60$

$$\begin{array}{rl} & x^2 - 6x + 9 + y^2 - 10y + 25 \\ + & x^2 - 8x + 16 + y^2 - 2y + 1 = 60 \\ \hline & 2x^2 - 14x + 25 + 2y^2 - 12y + 26 = 60 \end{array}$$

$2x^2 + 2y^2 - 14x - 12y + 51 - 60 = 0$

$2x^2 + 2y^2 - 14x - 12y - 9 = 0$ ……. Locus of P

02. find equation of the locus of the point P such that join of (–2,3) and (6,–7) subtends right angle at P

SOLUTION :

let P(x,y) be any point on the locus , A(–2,3) ; B(6,–7)

$PA^2 + PB^2 = AB^2$

$\left[(x+2)^2+(y-3)^2\right] + \left[(x-6)^2+(y+7)^2\right] = (-2-6)^2 + (3+7)^2$

$x^2 + 4x + 4 + y^2 - 6y + 9 + x^2 - 12x + 36 + y^2 + 14y + 49 = 64 + 100$

$2x^2 + 2y^2 - 8x + 8y + 13 + 85 = 164$

$2x^2 + 2y^2 - 8x + 8y + 98 - 164 = 0$

$2x^2 + 2y^2 - 8x + 8y - 66 = 0$

$x^2 + y^2 - 4x + 4y - 33 = 0$ ………. equation of the locus

03. the equation of the locus is $x^2 - 4x - 6y - 20 = 0$. If the origin is shifted to the point $(2,-4)$, axes remaining parallel , find the new equation of the locus

SOLUTION

origin shifted to $(h,k) \equiv (2,-4)$

old coordinates $\equiv (x,y)$, new coordinates $\equiv (X,Y)$

$$X = x - h \quad ; \quad Y = y - k$$
$$X = x - 2 \qquad Y = y + 4$$
$$x = X + 2 \qquad y = Y - 4$$

OLD equation of the locus : $x^2 - 4x - 6y - 20 = 0$

NEW equation of the locus : $(X + 2)^2 - 4(X + 2) - 6(Y - 4) - 20 = 0$

$$X^2 + 4X + 4 - 4X - 8 - 6Y + 24 - 20 = 0$$

$$X^2 - 6Y = 0$$

Q2. Attempt ALL FIVE of the following (10)

01.

Find the value of k for which points $P(1,-2)$, $Q(3,1)$ and $R(5,k)$ are collinear

$$m_{PQ} = \frac{1 + 2}{3 - 1} = \frac{3}{2}$$

$$m_{QR} = \frac{k - 1}{5 - 3} = \frac{k-1}{2}$$

Since P ,Q ,R are collinear ,

$m_{PQ} = m_{QR}$.

$$\frac{3}{2} = \frac{k - 1}{2}$$

$$3 = k - 1$$

$$\therefore k = 4$$

02.

find line passing through (2,5) and parallel to $3x - 4y - 7 = 0$

$3x - 4y - 7 = 0$

$$m = \frac{-a}{b} = -\frac{3}{-4} = \frac{3}{4}$$

Required Line

$m = 3/4$, passing through (2,5)

$$y - y_1 = m (x - x_1)$$

$$y - 5 = \frac{3}{4}(x - 2)$$

$$4y - 20 = 3x - 6$$

$$3x - 4y + 14 = 0$$

03.

find line whose x − intercept is 3 and which is perpendicular to line $3x - y + 23 = 0$

$3x - y + 23 = 0$

$m = \frac{-a}{b} = -\frac{3}{-1} = 3$

<u>Required Line</u>

$m = {}^{-1}/_{3}$, passing through (3,0)

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{-1}{3}(x - 3)$

$3y = -x + 3$

$x + 3y - 3 = 0$

04.

find the measure of acute angle between the lines $12x - 4y = 5$ & $4x + 2y = 7$

$12x - 4y = 5$

$m_1 = -\frac{a}{b} = -\frac{12}{-4} = 3$

$4x + 2y = 7$

$m_2 = -\frac{a}{b} = -\frac{4}{2} = -2$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1.m_2}\right|$

$= \left|\frac{3-(-2)}{1+3(-2)}\right|$

$= \left|\frac{5}{-5}\right|$

$= 1$

$\theta = 45°$

05.

find distance between $6x + 8y + 21 = 0$ & $3x + 4y + 7 = 0$

Line 1 : $6x + 8y + 21 = 0$

Line 2 : $\cancel{3x + 4y + 7 = 0}$

$6x + 8y + 14 = 0$

Distance between the two parallel lines :

$d = \left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$

$= \left|\frac{21 - 14}{\sqrt{6^2 + 8^2}}\right|$

$= \frac{7}{10}$

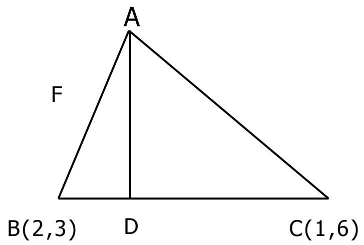
$= 0.7$ units

Q3. Attempt ANY THREE of the following (09)

01.

ΔABC . A(1,4) , B(2,3) , C(1,6) .

Find equations of altitudes AD , median BE



ALTITUDE AD

$$m_{BC} = \frac{6-3}{1-2} = -3$$

$$\therefore \; m_{AD} = {}^{1}/_{3} \qquad (AD \perp BC)$$

$m = {}^{1}/_{3}$, A(1,4)

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{1}{3}(x - 1)$$

$$3y - 12 = x - 1$$

$$x - 3y + 11 = 0$$

MEDIAN BE

$$E \equiv \left(\frac{1+1}{2}, \frac{4+6}{2}\right) \equiv (1,5)$$

B(2,3) , E(1,5)

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 3 = \frac{5-3}{1-2}(x - 2)$$

$$y - 3 = -2(x - 2)$$

$$2x + y - 7 = 0$$

02. Find equation of line which passes thro' point of intersection of lines $x+2y-3=0$ and $3x+4y-5=0$ and which is perpendicular to the line $x-3y+5=0$

Point of Intersection

$$x + 2y = 3 \quad \times 2$$

$$3x + 4y = 5$$

$$3x + 4y = 5$$

$$2x + 4y = 6$$

$$x = -1$$

subs in (1) $y = 2$ $\therefore (-1,2)$

$$x - 3y + 5 = 0$$

$$m = \frac{-a}{b} = \frac{-1}{-3} = \frac{1}{3}$$

Equation of required line

$m = -3$, passing through $(-1,2)$

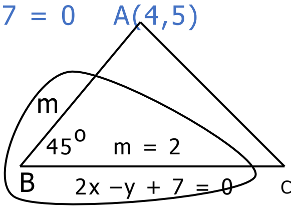
$$y - y_1 = m(x - x_1)$$

$$y - 2 = -3(x + 1)$$

$$3x + y + 1 = 0$$

03.

Find equations of the lines passing through the point (4,5) and making an angle of 45° with the line $2x - y + 7 = 0$



STEP 1 :

$2x - y + 7 = 0$

$m = \frac{-a}{b} = \frac{-2}{(-1)} = 2$

STEP 2 :

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1.m_2}\right|$$

$$\tan 45 = \left|\frac{m - 2}{1 + m(2)}\right|$$

$$1 = \left|\frac{m - 2}{1 + 2m}\right|$$

$\frac{m-2}{1+2m} = 1$	$\frac{m-2}{1+2m} = -1$
$m - 2 = 1 + 2m$	$m - 2 = -1 - 2m$
$m - 2m = 1 + 2$	$m + 2m = -1 + 2$
$-m = 3$	$3m = 2$
$m = -3$	$m = \frac{1}{3}$

STEP 3

Equation of AB : $m = -3$, A(4, 5)

$y - y_1 = m(x - x_1)$

$y - 5 = -3(x - 4)$

$y - 5 = -3x + 12$

$3x + y - 17 = 0$

Equation of AC : $m = \frac{1}{3}$, A(4, 5)

$y - y_1 = m(x - x_1)$

$y - 5 = \frac{1}{3}(x - 4)$

$3y - 15 = x - 4$

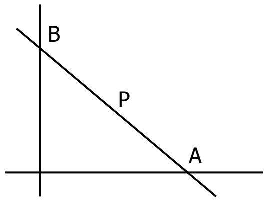
$x - 3y + 15 - 4 = 0$

$x - 3y + 11 = 0$

04.

Find the equation of line which passes through (1,2) and the midpoint of the portion of the line $3x - 4y + 24 = 0$ intercepted between the coordinate axes

SOLUTION



STEP 1 :

$3x - 4y + 24 = 0$

put y = 0 ; $3x + 24 = 0$

$3x = -24$ $\quad\therefore$ A (−8,0)

put x = 0 ; $-4y + 24 = 0$

$-4y = -24$ $\quad\therefore$ B (0.6)

STEP 2 :

P be the midpoint of AB

Using midpoint formula

$$P \equiv \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\equiv \left(\frac{-8 + 0}{2}, \frac{0 + 6}{2}\right)$$

$P \equiv (-4,3)$

STEP 3 :

Line is passing through (1,2) & (−4,3)

Equation of the line

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 2 = \frac{3 - 2}{-4 - 1}(x - 1)$$

$$y - 2 = \frac{1}{-5}(x - 1)$$

$-5y + 10 = x - 1$

$x + 5y - 11 = 0$